### Instructor's Manual

## **Intermediate Microeconomics**

**Ninth Edition** 

### Instructor's Manual

### **Intermediate Microeconomics**

**Ninth Edition** 

Instructor's Manual by Hal R. Varian

Answers to Workouts by Hal R. Varian and Theodore C. Bergstrom



W. W. Norton & Company • New York • London

Copyright © 2014, 2010, 2002, 1999, 1996, 1993, 1990, 1988 by W. W. Norton & Company

All rights reserved Printed in the United States of America

Typeset in  $T_EX$  by Hal R. Varian

NINTH EDITION

### ISBN 978-0-393-93676-6

W. W. Norton & Company, Inc., 500Fifth Avenue, New York, N.Y. 10110

W. W. Norton & Company, Ltd., Castle House, 75/76 Wells Street, London W1T 3QT

www.wwnorton.com

 $1\; 2\; 3\; 4\; 5\; 6\; 7\; 8\; 9\; 0$ 

### **CONTENTS**

## Part I Chapter Highlights

1. The Market	1
2. Budget Constraint	4
3. Preferences	7
4. Utility	10
5. Choice	13
6. Demand	16
7. Revealed Preference	18
8. Slutsky Equation	21
9. Buying and Selling	23
10. Intertemporal Choice	26
11. Asset Markets	29
12. Uncertainty	31
13. Risky Assets	33
14. Consumer's Surplus	35
15. Market Demand	37
16. Equilibrium	39
17. Measurement	42
18. Auctions	44
19. Technology	46
20. Profit Maximization	48
21. Cost Minimization	50
22. Cost Curves	52
23. Firm Supply	54
24. Industry Supply	56
25. Monopoly	59
26. Monopoly Behavior	61
27. Factor Markets	63
28. Oligopoly	65
29. Game Theory	68

30. Game Applications	71
31. Behavioral Economics	75
32. Exchange	78
33. Production	81
34. Welfare	83
35. Externalities	85
36. Information Technology	88
37. Public Goods	92
38. Information	95
Part II Answers to Workouts	
1. The Market	1
2. Budget Constraint	5
3. Preferences	17
4. Utility	33
5. Choice	49
6. Demand	67
7. Revealed Preference	81
8. Slutsky Equation	97
9. Buying and Selling	111
10. Intertemporal Choice	131
11. Asset Markets	147
12. Uncertainty	161
13. Risky Assets	175
14. Consumer's Surplus	181
15. Market Demand	191
16. Equilibrium	201

17. Measurement	219
18. Auctions	221
19. Technology	239
20. Profit Maximization	251
21. Cost Minimization	267
22. Cost Curves	279
23. Firm Supply	285
24. Industry Supply	295
25. Monopoly	311
26. Monopoly Behavior	317
27. Factor Markets	331
28. Oligopoly	335
29. Game Theory	351
30. Game Applications	365
31. Behavioral Economics	381
32. Exchange	391
33. Production	407
34. Welfare	417
35. Externalities	427
36. Information Technology	439
37. Public Goods	455
38. Asymmetric Information	469

# The Market

This chapter was written so I would have something to talk about on the first day of class. I wanted to give students an idea of what economics was all about, and what my lectures would be like, and yet not have anything that was really critical for the course. (At Michigan, students are still shopping around on the first day, and a good number of them won't necessarily be at the lecture.)

I chose to discuss a housing market since it gives a way to describe a number of economic ideas in very simple language and gives a good guide to what lies ahead. In this chapter I was deliberately looking for surprising results—analytic insights that wouldn't arise from "just thinking" about a problem. The two most surprising results that I presented are the condominium example and the tax example in Section 1.6. It is worth emphasizing in class just why these results are true, and how they illustrate the power of economic modeling.

It also makes sense to describe their limitations. Suppose that every condominium conversion involved knocking out the walls and creating two apartments. Then what would happen to the price of apartments? Suppose that the condominiums attracted suburbanites who wouldn't otherwise consider renting an apartment. In each of these cases, the price of remaining apartments would rise when condominium conversion took place.

The point of a simple economic model of the sort considered here is to focus our thoughts on what the relevant effects are, not to come to a once-and-for-all conclusion about the urban housing market. The real insight that is offered by these examples is that you have to consider both the supply and the demand side of the apartment market when you analyze the impact of this particular

The only concept that the students seem to have trouble with in this chapter is the idea of Pareto efficiency. I usually talk about the idea a little more than is in the book and rephrase it a few times. But then I tell them not to worry about it too much, since we'll look at it in great detail later in the course.

The workbook problems here are pretty straightforward. The biggest problem is getting the students to draw the true (discontinuous) demand curve, as in Figure 1.1, rather than just to sketch in a downward-sloping curve as in Figure 1.2. This is a good time to emphasize to the students that when they are given numbers describing a curve, they have to use the numbers—they can't just sketch in any old shape.

### The Market

- A. Example of an economic model the market for apartments
  - 1. models are simplifications of reality
  - 2. for example, assume all apartments are identical
  - 3. some are close to the university, others are far away
  - 4. price of outer-ring apartments is **exogenous** determined outside the model
  - 5. price of inner-ring apartments is **endogenous** determined within the model
- B. Two principles of economics
  - 1. **optimization principle** people choose actions that are in their interest
  - 2. **equilibrium principle** people's actions must eventually be consistent with each other
- C. Constructing the demand curve
  - 1. line up the people by willingness-to-pay. See Figure 1.1.
  - 2. for large numbers of people, this is essentially a smooth curve as in Figure 1.2.
- D. Supply curve
  - 1. depends on time frame
  - 2. but we'll look at the **short run** when supply of apartments is fixed.
- E. Equilibrium
  - 1. when demand equals supply
  - 2. price that clears the market
- F. Comparative statics
  - 1. how does equilibrium adjust when economic conditions change?
  - 2. "comparative" compare two equilibria
  - 3. "statics" only look at equilibria, not at adjustment
  - 4. example increase in supply lowers price; see Figure 1.5.
  - 5. example create condos which are purchased by renters; no effect on price; see Figure 1.6.
- G. Other ways to allocate apartments
  - 1. discriminating monopolist
  - 2. ordinary monopolist
  - 3. rent control
- H. Comparing different institutions
  - 1. need a criterion to compare how efficient these different allocation methods are.
  - 2. an allocation is **Pareto efficient** if there is no way to make some group of people better off without making someone else worse off.
  - 3. if something is *not* Pareto efficient, then there *is* some way to make some people better off without making someone else worse off.
  - 4. if something is not Pareto efficient, then there is some kind of "waste" in the system.
- I. Checking efficiency of different methods
  - 1. free market efficient
  - 2. discriminating monopolist efficient
  - 3. ordinary monopolist not efficient
  - 4. rent control not efficient

- J. Equilibrium in long run1. supply will change2. can examine efficiency in this context as well

# **Budget Constraint**

Most of the material here is pretty straightforward. Drive home the formula for the slope of the budget line, emphasizing the derivation on page 23. Try some different notation to make sure that they see the *idea* of the budget line, and don't just memorize the formulas. In the workbook, we use a number of different choices of notation for precisely this reason. It is also worth pointing out that the slope of a line depends on the (arbitrary) choice of which variable is plotted on the vertical axis. It is surprising how often confusion arises on this point.

Students sometimes have problems with the idea of a numeraire good. They understand the algebra, but they don't understand when it would be used. One nice example is in foreign currency exchange. If you have English pounds and American dollars, then you can measure the total wealth that you have in either dollars or pounds by choosing one or the other of the two goods as numeraire.

In the workbook, students sometimes get thrown in exercises where one of the goods has a negative price, so the budget line has a positive slope. This comes from trying to memorize formulas and figures rather than thinking about the problem. This is a good exercise to go over in order to warn students about the dangers of rote learning!

### **Budget Constraint**

- A. Consumer theory: consumers choose the best bundles of goods they can afford.
  - 1. this is virtually the entire theory in a nutshell
  - 2. but this theory has many surprising consequences
- B. Two parts to theory
  - 1. "can afford" budget constraint
  - 2. "best" according to consumers' **preferences**

- 1. test it see if it is adequate to describe consumer behavior
- 2. predict how behavior changes as economic environment changes
- 3. use observed behavior to estimate underlying values
  - a) cost-benefit analysis
  - b) predicting impact of some policy

### D. Consumption bundle

- 1.  $(x_1, x_2)$  how much of each good is consumed
- 2.  $(p_1, p_2)$  prices of the two goods
- 3. m money the consumer has to spend
- 4. budget constraint:  $p_1x_1 + p_2x_2 \leq m$
- 5. all  $(x_1, x_2)$  that satisfy this constraint make up the **budget set** of the consumer. See Figure 2.1.

### E. Two goods

- 1. theory works with more than two goods, but can't draw pictures.
- 2. often think of good 2 (say) as a composite good, representing money to spend on other goods.
- 3. budget constraint becomes  $p_1x_1 + x_2 \leq m$ .
- 4. money spent on good 1  $(p_1x_1)$  plus the money spent on good 2  $(x_2)$  has to be less than or equal to the amount available (m).

### F. Budget line

- 1.  $p_1x_1 + p_2x_2 = m$
- 2. also written as  $x_2 = m/p_2 (p_1/p_2)x_1$ .
- 3. budget line has slope of  $-p_1/p_2$  and vertical intercept of  $m/p_2$ .
- 4. set  $x_1 = 0$  to find vertical intercept  $(m/p_2)$ ; set  $x_2 = 0$  to find horizontal intercept  $(m/p_1)$ .
- 5. slope of budget line measures opportunity cost of good 1 how much of good 2 you must give up in order to consume more of good 1.

### G. Changes in budget line

- 1. increasing m makes parallel shift out. See Figure 2.2.
- 2. increasing  $p_1$  makes budget line steeper. See Figure 2.3.
- 3. increasing  $p_2$  makes budget line flatter
- 4. just see how intercepts change
- 5. multiplying all prices by t is just like dividing income by t
- 6. multiplying all prices and income by t doesn't change budget line
  - a) "a perfectly balanced inflation doesn't change consumption possibilities"

### H. The numeraire

- 1. can arbitrarily assign one price a value of 1 and measure other price relative to that
- 2. useful when measuring relative prices; e.g., English pounds per dollar, 1987 dollars versus 1974 dollars, etc.

### I. Taxes, subsidies, and rationing

- 1. quantity tax tax levied on units bought:  $p_1 + t$
- 2. value tax tax levied on dollars spent:  $p_1 + \tau p_1$ . Also known as ad valorem tax
- 3. subsidies opposite of a tax
  - a)  $p_1 s$
  - b)  $(1 \sigma)p_1$

### 6 Chapter Highlights

- 4. lump sum tax or subsidy amount of tax or subsidy is independent of the consumer's choices. Also called a head tax or a poll tax
- 5. rationing can't consume more than a certain amount of some good

### J. Example — food stamps

- 1. before 1979 was an ad valorem subsidy on food
  - a) paid a certain amount of money to get food stamps which were worth more than they cost
  - b) some rationing component could only buy a maximum amount of food stamps
- 2. after 1979 got a straight lump-sum grant of food coupons. Not the same as a pure lump-sum grant since could only spend the coupons on food.

# **Preferences**

This chapter is more abstract and therefore needs somewhat more motivation than the previous chapters. It might be a good idea to talk about relations in general before introducing the particular idea of preference relations. Try the relations of "taller," and "heavier," and "taller and heavier." Point out that "taller and heavier" isn't a complete relation, while the other two are. This general discussion can motivate the general idea of preference relations.

Make sure that the students learn the specific examples of preferences such as perfect substitutes, perfect complements, etc. They will use these examples many, many times in the next few weeks!

When describing the ideas of perfect substitutes, emphasize that the defining characteristic is that the slope of the indifference curves is constant, not that it is -1. In the text, I always stick with the case where the slope is -1, but in the workbook, we often treat the general case. The same warning goes with the perfect complements case. I work out the symmetric case in the text and try to get the students to do the asymmetric case in the workbook.

The definition of the marginal rate of substitution is fraught with "sign confusion." Should the MRS be defined as a negative or a positive number? I've chosen to give the MRS its natural sign in the book, but I warn the students that many economists tend to speak of the MRS in terms of absolute value. Example: diminishing marginal rate of substitution refers to a situation where the absolute value of the MRS decreases as we move along an indifference curve. The actual value of the MRS (a negative number) is increasing in this movement!

Students often begin to have problems with the workbook exercises here. The first confusion they have is that they get mixed up about the idea that indifference curves measure the directions where preferences are constant, and instead draw lines that indicate the directions that preferences are increasing. The second problem that they have is in knowing when to draw just arbitrary curves that qualitatively depict some behavior or other, and when to draw exact shapes.

Try asking your students to draw their indifference curves between five dollar bills and one dollar bills. Offer to trade with them based on what they draw. In addition to getting them to think, this is a good way to supplement your faculty salary.

### **Preferences**

- A. Preferences are relationships between bundles.
  - 1. if a consumer would choose bundle  $(x_1, x_2)$  when  $(y_1, y_2)$  is available, then it is natural to say that bundle  $(x_1, x_2)$  is preferred to  $(y_1, y_2)$  by this consumer.
  - 2. preferences have to do with the entire *bundle* of goods, not with individual goods.

### B. Notation

- 1.  $(x_1, x_2) \succ (y_1, y_2)$  means the x-bundle is **strictly preferred** to the y-bundle
- 2.  $(x_1, x_2) \sim (y_1, y_2)$  means that the x-bundle is regarded as **indifferent** to the y-bundle
- 3.  $(x_1, x_2) \succeq (y_1, y_2)$  means the x-bundle is at least as good as (preferred to or indifferent to) the y-bundle

### C. Assumptions about preferences

- 1. complete any two bundles can be compared
- 2. reflexive any bundle is at least as good as itself
- 3. transitive if  $X \succ Y$  and  $Y \succ Z$ , then  $X \succ Z$ 
  - a) transitivity necessary for theory of optimal choice

### D. Indifference curves

- 1. graph the set of bundles that are indifferent to some bundle. See Figure 3.1.
- 2. indifference curves are like contour lines on a map
- 3. note that indifference curves describing two distinct levels of preference cannot cross. See Figure 3.2.
  - a) proof use transitivity

### E. Examples of preferences

- 1. perfect substitutes. Figure 3.3.
  - a) red pencils and blue pencils; pints and quarts
  - b) constant rate of trade-off between the two goods
- 2. perfect complements. Figure 3.4.
  - a) always consumed together
  - b) right shoes and left shoes; coffee and cream
- 3. bads. Figure 3.5.
- 4. neutrals. Figure 3.6.
- 5. satiation or bliss point Figure 3.7.

### F. Well-behaved preferences

- 1. monotonicity more of either good is better
  - a) implies indifference curves have negative slope. Figure 3.9.
- 2. convexity averages are preferred to extremes. Figure 3.10.
  - a) slope gets flatter as you move further to right
  - b) example of non-convex preferences

### G. Marginal rate of substitution

- 1. slope of the indifference curve
- 2.  $MRS = \Delta x_2/\Delta x_1$  along an indifference curve. Figure 3.11.
- 3. sign problem natural sign is negative, since indifference curves will generally have negative slope
- 4. measures how the consumer is willing to trade off consumption of good 1 for consumption of good 2. Figure 3.12.

- 5. measures marginal willingness to pay (give up)
  - a) not the same as how much you have to pay
  - b) but how much you would be willing to pay

# Utility

In this chapter, the level of abstraction kicks up another notch. Students often have trouble with the idea of utility. It is sometimes hard for trained economists to sympathize with them sufficiently, since it seems like such an obvious notion to us.

Here is a way to approach the subject. Suppose that we return to the idea of the "heavier than" relation discussed in the last chapter. Think of having a big balance scale with two trays. You can put someone on each side of the balance scale and see which person is heavier, but you don't have any standardized weights. Nevertheless you have a way to determine whether x is heavier than y.

Now suppose that you decide to establish a scale. You get a bunch of stones, check that they are all the same weight, and then measure the weight of individuals in stones. It is clear that x is heavier than y if x's weight in stones is heavier than y's weight in stones.

Somebody else might use different units of measurements—kilograms, pounds, or whatever. It doesn't make any difference in terms of deciding who is heavier. At this point it is easy to draw the analogy with utility—just as pounds give a way to represent the "heavier than" order numerically, utility gives a way to represent the preference order numerically. Just as the units of weight are arbitrary, so are the units of utility.

This analogy can also be used to explore the concept of a positive monotonic transformation, a concept that students have great trouble with. Tell them that a monotonic transformation is just like changing units of measurement in the weight example.

However, it is also important for students to understand that nonlinear changes of units are possible. Here is a nice example to illustrate this. Suppose that wood is always sold in piles shaped like cubes. Think of the relation "one pile has more wood than another." Then you can represent this relation by looking at the measure of the sides of the piles, the surface area of the piles, or the volume of the piles. That is, x,  $x^2$ , or  $x^3$  gives exactly the same comparison between the piles. Each of these numbers is a different representation of the utility of a cube of wood.

Be sure to go over carefully the examples here. The Cobb-Douglas example is an important one, since we use it so much in the workbook. Emphasize that it is just a nice functional form that gives convenient expressions. Be sure to

elaborate on the idea that  $x_1^a x_2^b$  is the general form for Cobb-Douglas preferences, but various monotonic transformations (e.g., the log) can make it look quite different. It's a good idea to calculate the MRS for a few representations of the Cobb-Douglas utility function in class so that people can see how to do them and, more importantly, that the MRS doesn't change as you change the representation of utility.

The example at the end of the chapter, on commuting behavior, is a very nice one. If you present it right, it will convince your students that utility is an operational concept. Talk about how the same methods can be used in marketing surveys, surveys of college admissions, etc.

The exercises in the workbook for this chapter are very important since they drive home the ideas. A lot of times, students *think* that they understand some point, but they don't, and these exercises will point that out to them. It is a good idea to let the students discover for themselves that a sure-fire way to tell whether one utility function represents the same preferences as another is to compute the two marginal rate of substitution functions. If they don't get this idea on their own, you can pose it as a question and lead them to the answer.

### Utility

- A. Two ways of viewing utility
  - 1. old way
    - a) measures how "satisfied" you are
      - 1) not operational
      - 2) many other problems
  - 2. new way
    - a) summarizes preferences
    - b) a utility function assigns a number to each bundle of goods so that more preferred bundles get higher numbers
    - c) that is,  $u(x_1, x_2) > u(y_1, y_2)$  if and only if  $(x_1, x_2) \succ (y_1, y_2)$
    - d) only the ordering of bundles counts, so this is a theory of **ordinal utility**
    - e) advantages
      - 1) operational
      - 2) gives a complete theory of demand
- B. Utility functions are not unique
  - 1. if  $u(x_1, x_2)$  is a utility function that represents some preferences, and  $f(\cdot)$  is any increasing function, then  $f(u(x_1, x_2))$  represents the same preferences
  - 2. why? Because  $u(x_1, x_2) > u(y_1, y_2)$  only if  $f(u(x_1, x_2)) > f(u(y_1, y_2))$
  - 3. so if  $u(x_1, x_2)$  is a utility function then any positive monotonic transformation of it is also a utility function that represents the same preferences
- C. Constructing a utility function
  - 1. can do it mechanically using the indifference curves. Figure 4.2.
  - 2. can do it using the "meaning" of the preferences
- D. Examples
  - 1. utility to indifference curves
    - a) easy just plot all points where the utility is constant
  - 2. indifference curves to utility
  - 3. examples
    - a) perfect substitutes all that matters is total number of pencils, so  $u(x_1, x_2) = x_1 + x_2$  does the trick

- 1) can use any monotonic transformation of this as well, such as  $\log (x_1 + x_2)$
- b) perfect complements what matters is the minimum of the left and right shoes you have, so  $u(x_1, x_2) = \min\{x_1, x_2\}$  works
- c) quasilinear preferences indifference curves are vertically parallel. Figure 4.4.
  - 1) utility function has form  $u(x_1, x_2) = v(x_1) + x_2$
- d) Cobb-Douglas preferences. Figure 4.5.
  - 1) utility has form  $u(x_1, x_2) = x_1^b x_2^c$
  - 2) convenient to take transformation  $f(u)=u^{\frac{1}{b+c}}$  and write  $x_1^{\frac{b}{b+c}}x_2^{\frac{c}{b+c}}$
  - 3) or  $x_1^a x_2^{1-a}$ , where a = b/(b+c)

### E. Marginal utility

- 1. extra utility from some extra consumption of one of the goods, holding the other good fixed
- 2. this is a derivative, but a special kind of derivative a partial derivative
- 3. this just means that you look at the derivative of  $u(x_1, x_2)$  keeping  $x_2$  fixed treating it like a constant
- 4. examples
  - a) if  $u(x_1, x_2) = x_1 + x_2$ , then  $MU_1 = \partial u / \partial x_1 = 1$
  - b) if  $u(x_1, x_2) = x_1^a x_2^{1-a}$ , then  $MU_1 = \partial u/\partial x_1 = ax_1^{a-1} x_2^{1-a}$
- 5. note that marginal utility depends on which utility function you choose to represent preferences
  - a) if you multiply utility times 2, you multiply marginal utility times 2
  - b) thus it is not an operational concept
  - c) however, MU is closely related to MRS, which is an operational concept
- 6. relationship between MU and MRS
  - a)  $u(x_1, x_2) = k$ , where k is a constant, describes an indifference curve
  - b) we want to measure slope of indifference curve, the MRS
  - c) so consider a change  $(dx_1, dx_2)$  that keeps utility constant. Then

$$MU_1 dx_1 + MU_2 dx_2 = 0$$
$$\frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = 0$$

d) hence

$$\frac{dx_2}{dx_1} = -\frac{MU_1}{MU_2}$$

e) so we can compute MRS from knowing the utility function

### F. Example

- 1. take a bus or take a car to work?
- 2. let  $x_1$  be the time of taking a car,  $y_1$  be the time of taking a bus. Let  $x_2$  be cost of car, etc.
- 3. suppose utility function takes linear form  $U(x_1,\ldots,x_n)=\beta_1x_1+\ldots+\beta_nx_n$
- 4. we can observe a number of choices and use statistical techniques to estimate the parameters  $\beta_i$  that best describe choices
- 5. one study that did this could forecast the actual choice over 93% of the time
- 6. once we have the utility function we can do many things with it:
  - a) calculate the marginal rate of substitution between two characteristics
    - 1) how much money would the average consumer give up in order to get a shorter travel time?
  - b) forecast consumer response to proposed changes
  - c) estimate whether proposed change is worthwhile in a benefit-cost sense

# Choice

This is the chapter where we bring it all together. Make sure that students understand the *method* of maximization and don't just memorize the various special cases. The problems in the workbook are designed to show the futility of memorizing special cases, but often students try it anyway.

The material in Section 5.4 is very important—I introduce it by saying "Why should you *care* that the MRS equals the price ratio?" The answer is that this allows economists to determine something about peoples' trade-offs by observing market prices. Thus it allows for the possibility of benefit-cost analysis.

The material in Section 5.5 on choosing taxes is the first big non-obvious result from using consumer theory ideas. I go over it very carefully, to make sure that students understand the result, and emphasize how this analysis uses the techniques that we've developed. Pound home the idea that the analytic techniques of microeconomics have a big payoff—they allow us to answer questions that we wouldn't have been able to answer without these techniques.

If you are doing a calculus-based course, be sure to spend some time on the appendix to this chapter. Emphasize that to solve a constrained maximization problem, you must have two equations. One equation is the constraint, and one equation is the optimization condition. I usually work a Cobb-Douglas and a perfect complements problem to illustrate this. In the Cobb-Douglas case, the optimization condition is that the MRS equals the price ratio. In the perfect complements case, the optimization condition is that the consumer chooses a bundle at the corner.

#### Choice

### A. Optimal choice

- 1. move along the budget line until preferred set doesn't cross the budget set. Figure 5.1.
- 2. note that tangency occurs at optimal point necessary condition for optimum. In symbols:  $MRS = -\text{price ratio} = -p_1/p_2$ .
  - a) exception kinky tastes. Figure 5.2.
  - b) exception boundary optimum. Figure 5.3.
- 3. tangency is not sufficient. Figure 5.4.
  - a) unless indifference curves are convex.

- b) unless optimum is interior.
- 4. optimal choice is demanded bundle
  - a) as we vary prices and income, we get demand functions.
  - b) want to study how optimal choice the demanded bundle changes as price and income change

### B. Examples

- 1. perfect substitutes:  $x_1 = m/p_1$  if  $p_1 < p_2$ ; 0 otherwise. Figure 5.5.
- 2. perfect complements:  $x_1 = m/(p_1 + p_2)$ . Figure 5.6.
- 3. neutrals and bads:  $x_1 = m/p_1$ .
- 4. discrete goods. Figure 5.7.
  - a) suppose good is either consumed or not
  - b) then compare  $(1, m p_1)$  with (0, m) and see which is better.
- 5. concave preferences: similar to perfect substitutes. Note that tangency doesn't work. Figure 5.8.
- 6. Cobb-Douglas preferences:  $x_1 = am/p_1$ . Note constant budget shares, a = budget share of good 1.

### C. Estimating utility function

- 1. examine consumption data
- 2. see if you can "fit" a utility function to it
- 3. e.g., if income shares are more or less constant, Cobb-Douglas does a good job
- 4. can use the fitted utility function as guide to policy decisions
- 5. in real life more complicated forms are used, but basic idea is the same

### D. Implications of MRS condition

- 1. why do we care that MRS = -price ratio?
- 2. if everyone faces the same prices, then everyone has the same local trade-off between the two goods. This is independent of income and tastes.
- 3. since everyone locally values the trade-off the same, we can make policy judgments. Is it worth sacrificing one good to get more of the other? Prices serve as a guide to relative marginal valuations.

## E. Application — choosing a tax. Which is better, a commodity tax or an income tax?

- 1. can show an income tax is always better in the sense that given any commodity tax, there is an income tax that makes the consumer better off. Figure 5.9.
- 2. outline of argument:
  - a) original budget constraint:  $p_1x_1 + p_2x_2 = m$
  - b) budget constraint with tax:  $(p_1 + t)x_1 + p_2x_2 = m$
  - c) optimal choice with tax:  $(p_1 + t)x_1^* + p_2x_2^* = m$
  - d) revenue raised is  $tx_1^*$
  - e) income tax that raises same amount of revenue leads to budget constraint:  $p_1x_1 + p_2x_2 = m tx_1^*$ 
    - 1) this line has same slope as original budget line
    - 2) also passes through  $(x_1^*, x_2^*)$
    - 3) proof:  $p_1x_1^* + p_2x_2^* = m tx_1^*$
    - 4) this means that  $(x_1^*, x_2^*)$  is affordable under the income tax, so the optimal choice under the income tax must be even better than  $(x_1^*, x_2^*)$

### 3. caveats

a) only applies for one consumer — for each consumer there is an income tax that is better

- b) income is exogenous if income responds to tax, problems
- c) no supply response only looked at demand side
- F. Appendix solving for the optimal choice
  - 1. calculus problem constrained maximization
  - 2. max  $u(x_1, x_2)$  s.t.  $p_1x_1 + p_2x_2 = m$
  - 3. method 1: write down  $MRS = p_1/p_2$  and budget constraint and solve.
  - 4. method 2: substitute from constraint into objective function and solve.
  - 5. method 3: Lagrange's method
    - a) write Lagrangian:  $L = u(x_1, x_2) \lambda(p_1x_1 + p_2x_2 m)$ .
    - b) differentiate with respect to  $x_1, x_2, \lambda$ .
    - c) solve equations.
  - 6. example 1: Cobb-Douglas problem in book
  - 7. example 2: quasilinear preferences
    - a) max  $u(x_1) + x_2$  s.t.  $p_1x_1 + x_2 = m$
    - b) easiest to substitute, but works each way

## **Demand**

This is a very important chapter, since it unifies all the material in the previous chapter. It is also the chapter that separates the sheep from the goats. If the student has been paying attention for the previous 5 chapters and has been religiously doing the homework, then it is fairly easy to handle this chapter. Alas, I have often found that students have developed a false sense of confidence after seeing budget constraints, drift through the discussions of preference and utility, and come crashing down to earth at Chapter 6.

So, the first thing to do is to get them to review the previous chapters. Emphasize how each chapter builds on the previous chapters, and how Chapter 6 represents a culmination of this building. In turn Chapter 6 is a foundation for further analysis, and must be mastered in order to continue.

Part of the problem is that there is a large number of new concepts in this chapter: offer curves, demand curves, Engel curves, inferior goods, Giffen goods, etc. A list of these ideas along with their definitions and page references is often helpful just for getting the concepts down pat.

If you are doing a calculus-based course, the material in the appendix on quasilinear preferences is quite important. We will refer to this treatment later on when we discuss consumer's surplus, so it is a good idea to go through it carefully now.

Students usually have a rough time with the workbook problems. In part, I think that this is due to the fact that we have now got a critical mass of ideas, and that it has to percolate a bit before they can start brewing some new ideas. A few words of encouragement help a lot here, as well as drawing links with the earlier chapters. Most students will go back on their own and see what they missed on first reading, if you indicate that is a good thing to do. Remember: the point of the workbook problems is to show the students what they don't understand, not to give them a pat on the back. The role of the professor is to give them a pat on the back, or a nudge in the behind, whichever seems more appropriate.

#### Demand

A. Demand functions — relate prices and income to choices

- B. How do choices change as economic environment changes?
  - 1. changes in income
    - a) this is a parallel shift out of the budget line
    - b) increase in income increases demand **normal** good. Figure 6.1.
    - c) increase in income decreases demand **inferior** good. Figure 6.2.
    - d) as income changes, the optimal choice moves along the **income expansion path**
    - e) the relationship between the optimal choice and income, with prices fixed, is called the **Engel curve**. Figure 6.3.
  - 2. changes in price
    - a) this is a tilt or pivot of the budget line
    - b) decrease in price increases demand **ordinary** good. Figure 6.9.
    - c) decrease in price decreases demand Giffen good. Figure 6.10.
    - d) as price changes the optimal choice moves along the offer curve
    - e) the relationship between the optimal choice and a price, with income and the other price fixed, is called the **demand curve**

### C. Examples

- 1. perfect substitutes. Figure 6.12.
- 2. perfect complements. Figure 6.13.
- 3. discrete good. Figure 6.14.
  - a) reservation price price where consumer is just indifferent between consuming next unit of good and not consuming it
  - b)  $u(0,m) = u(1,m-r_1)$
  - c) special case: quasilinear preferences
  - d)  $v(0) + m = v(1) + m r_1$
  - e) assume that v(0) = 0
  - f) then  $r_1 = v(1)$
  - g) similarly,  $r_2 = v(2) v(1)$
  - h) reservation prices just measure marginal utilities
- D. Substitutes and complements
  - 1. increase in  $p_2$  increases demand for  $x_1$  substitutes
  - 2. increase in  $p_2$  decreases demand for  $x_1$  complements
- E. Inverse demand curve
  - 1. usually think of demand curve as measuring quantity as a function of price
    but can also think of price as a function of quantity
  - 2. this is the inverse demand curve
  - 3. same relationship, just represented differently

# Revealed Preference

This is a big change of pace, and usually a welcome one. The basic idea of revealed preference, as described in Section 7.1, is a very intuitive one. All I want to do in this chapter is give the students the tools to express that intuition algebraically.

I think that the material in Section 7.3, on recovering preferences, is very exciting. Start out with the idea of indirect revealed preference, as depicted in Figure 7.2. Point out that the optimization model allows us to predict how this person would behave when faced with a choice between  $(x_1, x_2)$  and  $(z_1, z_2)$ , even though we have never observed the person when faced with this choice! This is a big idea, and a very important one. Again, drive home how the economic model of optimization allows us to make strong predictions about behavior.

Figure 7.3 is the natural extension of this line of reasoning. Given the idea of revealed preference, and more importantly the idea of *indirect* revealed preference, we can determine the shape of underlying indifference curves from looking at choice data. I motivate this in terms of benefit-cost issues, but you could also choose to think about forecasting demand for products in a marketing survey, or similar applications.

Once students understand the idea of revealed preference, they can usually understand the Weak Axiom right away. However, they generally have difficulty in actually checking whether the Weak Axiom is satisfied by some real numbers. I added Section 7.5 for this reason; it just outlines one systematic way to check WARP. The students can omit this in their first reading, but they might want to come back to it when they start to do the exercises. If your students know a little computer programming, you might ask them to think about how to write a computer program to check WARP.

The same comments go for the treatment of the Strong Axiom and checking SARP. This is probably overkill, but I found that students couldn't really handle problem 7.5 in the workbook without some guidance about how to systematically check SARP. Speaking of the workbook, the problems in this section are really fun. I am especially fond of 7.6 and 7.7. Problem 7.9 had some wrong numbers in it in early printings of *Workouts*, so people with old books should be warned.

Finally, the material on index numbers is very worthwhile. Students here about price indices and cost-of-living indices all the time, so it's nice to describe the theory that lies behind these ideas.

### Revealed Preference

#### A. Motivation

- 1. up until now we've started with preference and then described behavior
- 2. revealed preference is "working backwards" start with behavior and describe preferences
- 3. recovering preferences how to use observed choices to "estimate" the indifference curves

### B. Basic idea

- 1. if  $(x_1, x_2)$  is chosen when  $(y_1, y_2)$  is affordable, then we know that  $(x_1, x_2)$  is at least as good as  $(y_1, y_2)$
- 2. in equations: if  $(x_1, x_2)$  is chosen when prices are  $(p_1, p_2)$  and  $p_1x_1+p_2x_2 \ge p_1y_1+p_2y_2$ , then  $(x_1, x_2) \succeq (y_1, y_2)$
- 3. see Figure 7.1.
- 4. if  $p_1x_1 + p_2x_2 \ge p_1y_1 + p_2y_2$ , we say that  $(x_1, x_2)$  is **directly revealed** preferred to  $(y_1, y_2)$
- 5. if X is directly revealed preferred to Y, and Y is directly revealed preferred to Z (etc.), then we say that X is **indirectly revealed preferred** to Z. See Figure 7.2.
- 6. the "chains" of revealed preference can give us a lot of information about the preferences. See Figure 7.3.
- 7. the information revealed about tastes by choices can be used in formulating economic policy

### C. Weak Axiom of Revealed Preference

- 1. recovering preferences makes sense only if consumer is actually maximizing
- 2. what if we observed a case like Figure 7.4.
- 3. in this case X is revealed preferred to Y and Y is also revealed preferred to X!
- 4. in symbols, we have  $(x_1, x_2)$  purchased at prices  $(p_1, p_2)$  and  $(y_1, y_2)$  purchased at prices  $(q_1, q_2)$  and  $p_1x_1+p_2x_2>p_1y_1+p_2y_2$  and  $q_1y_1+q_2y_2>q_1x_1+q_2x_2$
- 5. this kind of behavior is inconsistent with the optimizing model of consumer choice
- 6. the Weak Axiom of Revealed Preference (WARP) rules out this kind of behavior
- 7. WARP: if  $(x_1, x_2)$  is directly revealed preferred to  $(y_1, y_2)$ , then  $(y_1, y_2)$  cannot be directly revealed preferred to  $(x_1, x_2)$
- 8. WARP: if  $p_1x_1+p_2x_2 \ge p_1y_1+p_2y_2$ , then it must happen that  $q_1y_1+q_2y_2 \le q_1x_1+q_2x_2$
- 9. this condition can be checked by hand or by computer

### D. Strong Axiom of Revealed Preference

- 1. WARP is only a necessary condition for behavior to be consistent with utility maximization
- 2. Strong Axiom of Revealed Preference (SARP): if  $(x_1, x_2)$  is directly or indirectly revealed preferred to  $(y_1, y_2)$ , then  $(y_1, y_2)$  cannot be directly or indirectly revealed preferred to  $(x_1, x_2)$
- 3. SARP is a necessary and sufficient condition for utility maximization
- 4. this means that if the consumer is maximizing utility, then his behavior must be consistent with SARP
- 5. furthermore if his observed behavior is consistent with SARP, then we can always find a utility function that explains the behavior of the consumer as maximizing behavior.

- 6. can also be tested by a computer
- E. Index numbers
  - 1. given consumption and prices in 2 years, base year  $\boldsymbol{b}$  and some other year  $\boldsymbol{t}$
  - 2. how does consumption in year t compare with base year consumption?
  - 3. general form of a consumption index:

$$\frac{w_1 x_1^t + w_2 x_2^t}{w_1 x_1^b + w_2 x_2^b}$$

- 4. natural to use prices as weights
- 5. get two indices depending on whether you use period t or period b prices
- 6. Paasche index uses period t (current period) weights:

$$\frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

7. Laspeyres index uses period b (base period) weights:

$$\frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

8. note connection with revealed preference: if Paasche index is greater than 1, then period t must be better than period b:

$$\frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b} > 1$$

$$p_1^t x_1^t + p_2^t x_2^t > p_1^t x_1^b + p_2^t x_2^b$$

- c) so period t is revealed preferred to period b
- 9. same sort of thing can be done with Laspeyres index if Laspeyres index is less than 1, consumer is worse off

# Slutsky Equation

Most books talk about income and substitution effects, but then they don't do anything with the ideas. My view is that you have to give the student enough of an understanding of an idea to be able to compute with it; otherwise, why bother?

The Slutsky decomposition is an analytical tool that allows us to understand how demand changes when a price changes. It does this by breaking the total change in demand up into smaller pieces. The sign of the overall effect depends on the sign of the pieces, but the sign of the pieces is easier to determine.

I have used the Slutsky definition of substitution effect in this chapter. This is because it is much easier to compute examples using this definition. The Hicksian definition is theoretically more elegant, but students can't compute with it until they have more advanced mathematical tools.

A large part of getting this material across is just convincing the students to read the book. The change in income necessary to compensate for a change in price is neither a difficult concept nor a difficult calculation, but it has to be repeated a few times before the students grasp it.

One way to describe income and substitution effects is to give an example based on their own consumption patterns. Talk about a student who spends all of her allowance on food and books. Suppose that the price of books drops in half, but her parents find out about it and cut her allowance. How much do they cut her allowance if they want her to keep her old consumption level affordable?

Once they grasp the idea of the substitution and income effect, it isn't hard to put them together in Section 8.4. The next real hurdle is expressing the Slutsky equation in terms of rates of change, as is done in Section 8.5. This is the way that we usually refer to the Slutsky equation in later chapters, so it is worthwhile going through the algebra so they can see where it comes from. However, if you don't want to go through the algebraic computations, just make sure that they get the basic point: the change in demand can be decomposed into a substitution effect (always negative, i.e., opposite the direction of price change) and an income effect (positive or negative depending on whether we have a normal or inferior good).

I usually skip the Optional sections in this chapter, but they are there for reference if needed. I like the tax rebate section, but it is a little sophisticated. Emphasize the idea that even if you give the money from the tax back to the

consumers, the demand for the good will go down and consumers will be left worse off.

### **Slutsky Equation**

- A. We want a way to decompose the effect of a price change into "simpler" pieces.
  - 1. that's what analysis is all about
  - 2. break up into simple pieces to determine behavior of whole
- B. Break up price change into a **pivot** and a **shift** see Figure 8.2.
  - 1. these are hypothetical changes
  - 2. we can examine each change in isolation and look at sum of two changes
- C. Change in demand due to pivot is the **substitution effect**.
  - 1. this measures how demand changes when we change prices, keeping purchasing power fixed
  - 2. how much would a person demand if he had just enough money to consume the original bundle?
  - 3. this isolates the pure effect from changing the relative prices
  - 4. substitution effect must be negative due to revealed preference.
    - a) "negative" means quantity moves opposite the direction of price
- D. Change in demand due to shift is the **income effect**.
  - 1. increase income, keep prices fixed
  - 2. income effect can increase or decrease demand depending on whether we have a normal or inferior good
- E. Total change in demand is substitution effect plus the income effect.
  - 1. if good is normal good, the substitution effect and the income effect reinforce each other
  - 2. if good is inferior good, total effect is ambiguous
  - 3. see Figure 8.3.
- F. Specific examples
  - 1. perfect complements Figure 8.4.
  - 2. perfect substitutes Figure 8.5.
  - 3. quasilinear Figure 8.6.
- G. Application rebating a tax
  - 1. put a tax on gasoline and return the revenues
  - 2. original budget constraint:  $px^* + y^* = m$
  - 3. after tax budget constraint: (p+t)x' + y' = m + tx'
  - 4. so consumption after tax satisfies px' + y' = m
  - 5. so (x', y') was affordable originally and rejected in favor of  $(x^*, y^*)$
  - 6. consumer must be worse off
- H. Rates of change
  - 1. can also express Slutsky effect in terms of rates of change
  - 2. takes the form

$$\frac{\partial x}{\partial p} = \frac{\partial x^s}{\partial p} - \frac{\partial x}{\partial m}x$$

3. can interpret each part just as before